VOLUME 79 SEPARATE No. 283

PROCEEDINGS

AMERICAN SOCIETY
OF
CIVIL ENGINEERS

SEPTEMBER, 1953

FLOCCULATION AND FLOCCULATION BASINS

by Thomas R. Camp, M. ASCE

SANITARY ENGINEERNG DIVISION

{Discussion open until January 1, 1954}

Headquarters of the Society
33 W. 39th St.
New York 18, N. Y.

PRICE $0.50 PER COPY

THIS PAPER

--represents an effort of the Society to deliver technical data direct from the author to the reader with the greatest possible speed. To this end, it has had none of the usual editing required in more formal publication procedures.

Readers are invited to submit discussion applying to current papers. For this paper the final closing dead line appears on the front cover.

Those who are planning papers or discussions for "Proceedings" will expedite Division and Committee action measurably by first studying the printed directions for the preparation of ASCE technical papers. For free copies of these directions—describing style, content, and format—address the Manager, Technical Publications, ASCE.

This paper was published at 1745 S. State Street, Ann Arbor, Mich., by the American Society of Civil Engineers. Editorial and General Offices are at 33 West Thirty-ninth Street, New York 18, N. Y.

FLOCCULATION AND FLOCCULATION BASINS

Thomas R. Camp[1], M. ASCE

Synopsis

An important process in the treatment of water, sewage and industrial wastes is the formation of suspended flocs which can be effectively removed from the liquid by settling or filtration. This process is known as flocculation or coagulation. Prior to about 1920, the nature of the process was not well understood by sanitary engineers and was frequently confused with mixing, the purpose of which is to distribute the coagulating chemicals in the liquid being treated so as to promote solution of the chemicals and completion of the chemical reactions. It has since been learned that flocculation is a physical process requiring gentle turbulence and time. Little progress has been made, however, in the scientific development of the principles involved and their application to design.

The fundamental theory of the physical process of the floc formation is presented in this paper together with evidence of experimental verification. The paper also includes methods of procedure in the practical application of the theory to the design of flocculation apparatus and basins. The symbols used in the paper are defined as they are introduced, and the definitions are tabulated in the appendix, Table 3.

Theory of Flocculation

The completion of the chemical reactions in a coagulation or softening process is almost instantaneous, after the chemicals are fully dissolved. The precipitates formed by the chemical reactions are first crystals of molecular size. The initial increase in size of these colloidal crystals is brought about by true diffusion or Brownian Motion until the size of the particles becomes too great to be influenced by Brownian motion. At this point the particles are still too small to be seen by the naked eye. The completion of the coagulation process requires gentle turbulent mixing of the suspension. It has been shown by Camp, Root and Bhoota (Jour. A.W.W.A. p.1913,1940) that the Browian Motion phase of coagulation is completed in a few seconds and is therefore of negligible importance in fixing tank dimensions as compared to the turbulent mixing phase.

It has been previously demonstrated (Camp, T.R. and Stein, P.C. "Velocity Gradients and Internal Work in Fluid Motion", Journal of the Boston Society of Civil Engineers, October 1943, also, Camp, T.R. "Sedimentation in the Design of Settling Tanks", Transactions, ASCE, 1946, p.920) that the rate of flocculation at a point in a fluid which is caused by the motion of the fluid is directly proportional to the absolute velocity gradient or space rate of change of velocity at the point and is directly proportional to the concentration of flocculable particles at the point. This relation has been independ-

1. Camp, Dresser & McKee, Consulting Engineers, Boston, Mass.

ently developed by others (Manley, R. St. J. and Mason, S. G. "Particle Motions in Sheared Suspensions. II, Collisions of Uniform Spheres", Journal of Colloid Science, Vol. 7, No. 4, August 1952, pp. 354-369) and experimental verification has been obtained with small glass spheres having an average diameter of 137 microns suspended in high viscosity corn syrup. The experimental verification was obtained in streamline motion of the fluid.

It has also been demonstrated (Camp, T. R. and Stein, P. C., previous reference) that the absolute velocity gradient at a point in a fluid in motion is equal to the square root of the ratio or the power loss by shear per unit of volume of fluid to the viscosity of the fluid. The velocity gradients at any moment throughout a vessel or chamber in use for flocculation vary considerably in magnitude being greatest at the solid boundaries of the paddles or other devices being used to introduce the mixing motion and being least in the corners of the chamber farthest removed from the point of introduction of the motion. Also the velocity gradients in a conduit, in which coagulation is taking place by the flow of the fluid, are greatest at the walls. Under steady conditions of work input, however, there is a mean velocity gradient which corresponds with the mean value of the rate of power dissipation throughout the tank or conduit.

The rate of power dissipation, i. e., the work of shear per unit of volume per unit of time at a point, is known as the "dissipation function" (stokes, G. G. "On the Theories of Internal Friction of Fluids in Motion, Etc.", Cambridge Philosophical Transactions, 1845). The mean value of the dissipation function, here designated as W, is equal to the total power dissipation divided by the volume of the chamber or conduit. The root mean square velocity gradient then is defined by the following relation:

$$G = \sqrt{\frac{W}{\mu}} \tag{1}$$

in which G is the root-mean-square velocity gradient in the chamber and μ is the absolute viscosity of the fluid.

Since the rate of floc formation is directly proportional to the velocity gradient G, it should follow that the greater the magnitude of G the less should be the time required to form the floc. Hence, for economy in the size of flocculation chambers, the velocity gradients should be made as large as practicable. The practical limit of the velocity gradient for any flocculation process is determined by the size of floc particles required, because there is a maximum size of floc particle associated with each velocity gradient as shown below.

The viscosity of a fluid is the proportionality constant between the unit shearing force, τ, and the velocity gradient at a point, G':

$$\tau = \mu G' \tag{2}$$

It is evident from equation (2) that the higher the velocity gradients become the greater are the shearing forces in a fluid. As floc particles grow in size they become weaker and are more easily sheared apart. Thus to form small floc particles, relatively high velocity gradients may be used; while for large floc particles, lower velocity gradients are required.

Experience in water and sewage treatment plants shows that too violent a mix in flocculation chambers will prevent the formation of floc particles

which are large enough to settle out effectively. Microscopic studies of flow in rapid sand water filters by Stein (Doctor's thesis, Massachusetts Institute of Technology, 1940) in which the computed value of the mean velocity gradient was approximately 200 sec^{-1} showed that no appreciable flocculation was taking place in the pores. Experiments by Hubley, Robertson and Mason (Hubley, C. E., Robertson, A. A. and Mason, S. G., "Flocculation in Suspensions of Large Particles", Canadian Journal of Research B, 28, December, 1950, p. 770-787) showed that suspensions of aged latex particles in water had very little tendency to flocculate at velocity gradients exceeding 10 sec^{-1}, whereas suspensions of wood pulp fibers required velocity gradients in excess of 50 sec^{-1} to break up the flocs appreciably. Experiments by Stein (previous reference) on the formation of hydrous ferric oxide floc in laboratory jar tests indicated that 20 sec^{-1} was optimum for flocculation and 40 sec^{-1} disrupted the flocs.

Since data regarding limiting velocity gradients are extremely meager in the field of water and sewage treatment, an effort has been made to throw some light on the matter by computations of the mean velocity gradients available in flocculation basins of existing water treatment plants. For this purpose, the data in ASCE Manual No. 19 "Water Treatment Plant Design" Table 2 were used. Table 2 of the manual contains information on 42 flocculation basins. The velocity gradients have been computed for twenty of these basins, omitting those on which the data were incomplete or for which the detention period was less than 10 minutes. The results of the computations are shown in Table 1. The velocity gradients in Table 1 are based on a water temperature of 50° F., at which the absolute viscosity is 0.273×10^{-4} lb sec per sq ft. The values of W are expressed in ft lbs per sec per cu ft.

It should be noted that the velocity gradients given in Table 1 are those at the capacity of the plant. The values for the dissipation function W are based on the capacity of the plants as shown in Column 1, and the head loss or horsepower as shown in Column 6. It is probable that even at plant capacity the water horsepower is much less than the connected horsepower. For the baffled basins, the value of W has been based on the retention periods and head losses as shown in columns 5 and 6 respectively, which are for the plant capacity. At lesser rates the head loss will be less and the retention period will be greater. Thus it may be concluded that the range of values for the mean velocity gradients as shown in Table 1 represent maxima and the optimum values are probably much smaller.

Since the rate of flocculation varies directly with the magnitude of the mean velocity gradient G, it follows that satisfactory flocculation should be produced with particular values of the product GT, where T is the flocculation period. If G and T are expressed in seconds, the product GT is a dimensionless number. The values of GT for the flocculation basins in Table 1 are shown in Column 10. Table 1 indicates that the maximum velocity gradients used in American practice range from about 20 to 74 sec^{-1}, and that the values of the product GT at plant capacity range from about 23,000 to 210,000. It may be assumed that these values represent coagulation basins which are operating with some degree of satisfaction.

Since, as pointed out above, higher velocity gradients may be used for small floc than for large floc particles, best economy should result where flocculation is carried out in several stages in a series of tanks with the velocity gradients progressively decreased as the floc particles grow in size. This procedure was first developed by Langelier, and is known as the Langelier Process. The sum of the GT values of the series of basins should be the

same as for a single basin to obtain satisfactory floc, but the detention period, and hence the cost for the series, should be less.

Means for Producing Velocity Gradients

The earliest flocculation basins were the so-called baffled mixing chambers with over-and-under or around-the-end baffles. Such basins were usually designed with a channel velocity from 0.3 to about 1.0 fps. For baffled mixing channels or conduits of any type, the value of the dissipation function W may be computed from the discharge and head loss as follows:

$$W = \frac{Q\gamma h_f}{V} = \frac{\gamma h_f}{T} \qquad (3)$$

where Q = the discharge
γ = the unit weight of the fluid
h_f = the head loss
V = the volume of the conduit or chamber
T = the retention period in sec.

Baffled mixing chambers are not very satisfactory for flocculation purposes because most of the head loss occurs at the 180° bends such that the velocity gradients are too high at the bends and not high enough in the straight channels. Another objection to this type of flocculation basin is that the magnitude of the dissipation function and hence the magnitude of the velocity gradients is directly proportional to the rate of discharge and cannot be varied at will by the operator. In modern plants mechanical stirring devices are used in which case the velocity gradients are independent of the rate of discharge. With mechanical stirring devices the velocity gradient may be changed by the operator if variable speed drives are provided. Flocculation may also be accomplished by means of diffused air in which case the magnitude of the mean velocity gradient may be controlled by the quantity of air used.

In some treatment plants turbulent mixing for coagulation has been induced by bringing the water into a basin, circular or square in plan, tangentially at high velocity. In such basins a spiral motion is induced. The value of the dissipation function W may be computed for such basins by means of Equation 3 with h_f taken as the velocity head at the inlet. In such basins the turbulence is not uniformly distributed since it is introduced at only one point. Basins of this type are not satisfactory in series because the floc will be broken up in passage from one basin to the next.

When turbulence is induced by mechanical mixing or by air, the value of the dissipation function W may be estimated from the drag force of the stirring blades of air bubbles in the water and the distance moved per second. The drag force for any body immersed in still water is given by Newton's law as follows:

$$F_D = C_D A \gamma \frac{v^2}{2g} \qquad (4)$$

Where C_D = the drag coefficient
A = the cross sectional area of the submerged object perpendicular to the direction of motion.

v = the relative velocity of the object with respect to the fluid

The distance moved by the submerged blade or air bubble per second is equal to its velocity with respect to the liquid. Unfortunately, with most types of stirring devices, the relative velocity of the stirring blade and liquid immediately surrounding the blade is difficult to determine. With rotary stirring devices the liquid is set in spiral motion with an average angular velocity less than the velocity of the rotors and with a velocity beyond the central core diminishing somewhat in magnitude with distance from the center. With reciprocating agitators, such as the so-called "walking beam" type, the velocity of the blade with respect to the tank varies throughout the stroke. During the stroke, motion is induced in the surrounding liquid. Since the direction of stroke is changed periodically, the effect is to retard the motion of the liquid. Nevertheless, the relative motion of blade and tank is not the same as the relative motion of the blade and surrounding liquid. Rising air bubbles also tend to impart motion to the surrounding liquid which in turn will result in faster rise of the air bubbles with respect to the tank.

Effect of Short-Circuiting

In order to facilitate the design of mechanical stirring mechanisms, most tanks with paddles mounted on vertical shafts are made circular or approximately square in plan and most tanks with paddles mounted on horizontal shafts are made approximately square in cross-sectional area. The effect of the mixing process is to short-circuit some of the water which enters the tank very quickly to the outlet port. This portion of the liquid will be inadequately coagulated.

The extent of short-circuiting in a tank may be measured experimentally by introducing a slug of tracer, such as dye or salt, at the inlet port and measuring its concentration after various intervals of time at the outlet port. If the mixing is so violent as to accomplish the dispersion of the slug of tracer uniformly throughout the contents of the tank instantaneously, the concentration of the tracer at the outlet may be computed by means of the following equation:

$$\frac{C}{C_0} = e^{\frac{-t}{T}} \tag{5}$$

where C = concentration of tracer at outlet after time t

C_0 = initial concentration if tracer is instantaneously dispersed throughout contents of tank

T = retention period of tank

e = base of Napierian logs

Fig. 1 is a dimensionless plot of Equation 5. Also shown on Fig. 1 is an experimental curve determined from a model of a cubical-shaped mixing tank with a moderate degree of mixing. The area under the curve represents the tracer slug. It will be noted from both the theoretical and experimental curves that a large part of the tracer is passed through the tank in less than the theoretical retention period. In the case of the theoretical curve, 40% of the slug leaves the tank in less than one-half the retention period, and 22% in less than one-fourth the dentention period.

In order to compensate for this short-circuiting several tanks should be placed in series so that the portion of the liquid which passes through the

upstream tank in a short period will have a chance to stay in the downstream tanks for a longer period.

Stein (unpublished paper) has developed the equation for the instantaneous dispersion curve for n tanks of equal size in series as follows:

$$\frac{C}{C_0} = \frac{n^n}{(n-1)!}\left(\frac{t}{T}\right)^{n-1} e^{-\frac{nt}{T}} \qquad (6)$$

where C_0 and T are based on the total volume of n tanks in series.

Fig. 2 shows the theoretical curves for one, two, three, four and ten instantaneous dispersion tanks in series. A study of the curves shows that with three tanks in series about 4% will pass through in one-fourth the retention period and about 19% in one-half the retention period.

Flocculation By Diffused Air Aeration

The work done on water in a flocculation tank by means of rising air bubbles may be computed either from the work of expansion of the air in the bubbles or from the drag forces between the air bubbles and the liquid. The results will be approximately the same whether expansion is assumed to be adiabatic or isothermal and the results obtained from expansion of the gas are approximately the same as those computed from the drag forces.

The work done by the drag forces on air bubbles in still water is equal to the drag force times the distance traveled by the bubbles (i.e. the depth of the diffusers below the water surface). The drag force is substantially equal to the weight of the water displaced by the air bubbles. Hence, the value of the dissipation function computed from the drag force is given by:

$$W = \frac{62.4\ Q_a}{V} \frac{34H}{\frac{H}{2}+34} \text{ ft. lbs. per sec per cu. ft. in water}$$

where Q_a = quantity of free air in cfs
V = volume of tank
H = depth of diffusers

$$Q_a \frac{34}{\frac{H}{2}+34} = \text{quantity of air at depth } \frac{H}{2} \text{ in cfs} \qquad (7)$$

$$62.4 Q_a \frac{34}{\frac{H}{2}+34} = \text{the drag force in lbs per sec.}$$

If the work is computed by isothermal expansion of the air as the bubbles rise through the depth H, the value of the dissipation function is given by the following:

$$W = \frac{4890\ Q_a}{V} \log_e \frac{H\ 34}{34} \quad \text{ft lbs per sec per cu ft in water} \qquad (7a)$$

For a flocculation tank of volume V the quantity of diffused air required in cubic feet per second for any mean velocity gradient G, is given by equations 8 and 8a below.

From the drag force:

$$Q_a = \frac{G^2 \mu V}{2120 \frac{H}{\frac{H}{2}+34}} \tag{8}$$

From isothermal expansion:

$$Q_a = \frac{G^2 \mu V}{4890 \log \frac{H+34}{34}} \tag{8a}$$

It will be noted from the above equations that the quantity of air required to produce a given mean velocity gradient is independent of the bubble size. For good flocculation, air bubbles should be distributed uniformly throughout the volume V, and should be small enough so that the velocity gradients in the immediate vicinity of the bubbles is not high enough to disrupt the floc. Since the regions of high velocity gradient with diffused air are much more numerous and more widely dispersed than with paddle flocculators, this latter precaution is of great importance.

It may be shown that individual bubbles rising in still water will conform to stokes' law up to bubble diameters of about 1.4 millimeters. The rising velocity of each bubble with respect to the water surrounding it is given by Stokes' law as shown below:

Stokes' law:

$$v = \frac{1}{18}\frac{g}{\mu}(\rho_1 - \rho) D^2 \tag{9}$$

where v = rising velocity
g = gravity constant
ρ_1 = mass density of air in bubble
ρ = mass density of water
and D = diameter of bubble

In the Stokes' law region the maximum value of the velocity gradient G'_{max} occurs at the interface between the bubble and the water and is given by the following relation:

$$G'_{max} = \frac{1}{6}\frac{g}{\mu}(\rho_1 - \rho) D \tag{10}$$

The sizes of air bubbles corresponding with various values of the maximum velocity gradient as computed from Equation 10 are shown in Table 2.

TABLE 2. AIR BUBBLE SIZES CORRESPONDING WITH VARIOUS VALUES OF MAXIMUM VELOCITY GRADIENT (WATER AT 50$_0$F.)

G'_{max}, Sec^{-1}	Bubble diameter, D. mm.
100	0.08
200	0.16
500	0.40
1000	0.80
2000±	2.0

It is apparent from Table 2 that bubble sizes must be small to prevent destruction of the floc in the final compartment of a flocculation basin where a mean velocity gradient of about 10 is required. In a typical case with diffusers at a depth of 14 ft, the quantity of air required at 50°F. water temperature for G equals 10 is estimated from Equation 8 at 0.00315 cfm per square foot of tank area. This is a very small quantity of air in terms of the capacity of porous plate diffusers now on the market.

If in the above example the size of the air bubbles is 2 millimeters, a common size for commercially available porous plates, the number of bubbles required per second per square foot of tank area is only about 300 and the bubbles must be spaced about 1.5 inches apart. In this case there will be velocity gradients in the order of magnitude of 2000 sec^{-1} at intervals of about 1.5 inches throughout the tank.

If in the above example the bubble size is made 0.16 millimeters so as to limit the maximum velocity gradient to about 200 sec^{-1}, the number of bubbles per second per square foot of tank area is estimated at about 5830 and the bubles will be spaced about 1/4 inch apart.

It is evident from the above discussion that a great deal of research and development will be required to obtain satisfactory diffusers so that flocculation by aeration may be controlled within optimum limits. Experience indicates that with air flocculation most of the floc particles will be carried upward by the air bubbles to form a scum at the surface of the water. This will reduce the concentration of flocculable particles and will thus increase the time required for flocculation. On the other hand, the floc can be removed by skimming at the surface of the aeration tank rather than by sedimentation in a separate unit following coagulation. Flocculation by aeration, therefore, offers an opportunity for a saving in first cost of plant. Attempts are now being made to exploit this advantage commercially.

Flocculation by Rotating Blades

Most of the commercially available apparatus for flocculation is of the revolving paddle type with either vertical or horizontal shafts. In most installations only rotor paddles are provided, and the only resistance there is to rotation of the water with the paddles is the drag on the walls of the tank. It is known that the water does move with the paddles, but little is known about the relative velocity of the paddles and the water.

In some existing plants the paddle widths are so great that the water in front of the paddles is carried along with the paddles at substantially the velocity of the paddles. The only mixing in such cases is provided between the outside edges of the paddles and the walls of the tank.

Bean (Elwood L. Bean, "Study of Physical Factors Affecting Flocculation", Water Works Engineering, January, 1953) states that the area of the paddles in a basin without stator blades should not be greater than 15 to 20 % of the cross-sectional area of a basin if rolling of the water is to be prevented. Bean further states that if the paddle area is 25% of the cross-sectional area, major rotation of the water will result. No evidence has been presented that the relative speed between the water and the paddles can be maintained proportional to the shaft speed unless stators are used to stabilize the motion.

In order to change the magnitude of the mean velocity gradient in an existing basin provided with rotor paddles, it is necessary to change the speed of rotation. For effective control of velocity gradient, the power dissipation must bear a definite relation with the speed of rotation after equilibrium is

is reached following starting of the paddles. In order to simplify design, it is desirable that the rotor paddles be mounted parallel with the shaft on arms attached to the shaft. The velocity of a paddle with respect to the tank is then proportional to its distance from the shaft.

The value of the dissipation function for a single rotor blade at distance r from the center of the shaft is given by the following equation:

$$W = \frac{F_D v}{V} = \frac{C_D A \frac{62.4}{64.4} v^3}{V} = \frac{C_D A \frac{62.4}{64.4}\left[2\pi r(1-k)S_s\right]^3}{V}$$

$$= \frac{\frac{62.4}{64.4}(2\pi)^3 C_D A r^3 (1-k)^3 S_s{}^3}{V} = \frac{239 C_D A r^3 (1-k)^3 S_s{}^3}{V} \qquad (11)$$

where v — velocity of blade with respect to the water
r — distance of center of blade from shaft
S_s — speed of shaft in RPS
kS_s — speed of water in RPS

The validity of equation (11) depends upon the existence of a constant ratio between the rotating speed of the shaft and of the water surrounding the paddle. In the Cambridge, Massachusetts, water treatment plant, where stators have been used, measurements show that the assumption is valid and that the power dissipation is directly proportional to the cube of the shaft speed.

A number of rotor paddles are required for a flocculation tank with varying distances out from the shaft and with varying paddle areas. The value of the dissipation function W for all the paddles in a tank is given by equation (11a) as follows:

$$W = \frac{239 C_D (1-k)^3 S_s{}^3}{V} \sum Ar^3 \qquad (11a)$$

where $\sum Ar^3$ = the sum of the values of Ar^3 for all rotors

It will be noted from equations (11) and (11a) that if the rotors start at full speed, the starting power will be much greater than the power after the water rotates at its equilibrium velocity. At the start the value of k is O. If, for example, the speed of the water after equilibrium is 1/2 the speed of the shaft with a value of k of 0.5, the starting power will be eight times the equilibrium power. Drive motors must be greatly oversized therefore, or provisions must be made to bring the paddles up to speed very slowly during starting.

Fig. 3 shows a half plan and sections of four new flocculation chambers recently constructed for the Cambridge, Massachusetts, Water Works. This plant is the first practical application of the theory of flocculation described in this paper. Stators have been provided in each compartment with a value of ΣAr^3 about equal to the value of ΣAr^3 for the rotors. Since nothing was known about the relative velocity of the rotors and the surrounding water and of the amount of power loss to be expected in the transmission, bearings and stuffing boxes, provisions were made in the specifications for a test to evaluate the unkowns after the first chamber was equipped with the flocculating mechanisms and for changing the size of sprocket wheels.

It will be noted from the figure that there are two shafts for driving the flocculating rotors in each chamber: a high-speed shaft for the smaller compartments and a low-speed shaft for the larger compartments. With sprockets finally selected, the speed of the high-speed shaft is 78% greater than the speed of the low-speed shaft. The mechanism for each chamber is driven by a single 10 HP variable speed electric motor of the movable brush type, having a three-to-one speed range. The minimum speed of the motor is approximately 833 rpm, and after testing, the stop on the speed change was set to limit the top speed at 2200 rpm.

The results of the tests on the flocculator mechanism at Cambridge are shown by the curves in Fig. 4. The tests showed that approximately 1 HP is lost through transmission, stuffing boxes and bearings, regardless of the speed. The tests also showed that after equilibrium is established the water revolves at about 24% of the speed of the rotors in the high-speed compartment and about 32% of the speed of the rotors in the low-speed compartments (that is, the values of k were found to be 0.24 and 0.32).

The speed of the low-speed shaft at Cambridge can be adjusted from about 1.1 rpm to about 2.9 rpm, and the speed of the high-speed shaft can be adjusted from about 2 rpm to about 5.2 rpm. The mean velocity gradient in the last compartment at minimum speed ranges from about 5 in the wintertime to about 7.6 in the summer and can be increased about 4.3-fold by increase in speed. For any speed and water temperature, the mean velocity gradients in the first three compartments are respectively 5.6, 3.8 and 1.9 times the mean velocity gradient in the fourth compartment.

Flocculation by Reciprocating Blades

An analysis of the work input in a flocculation tank with reciprocating paddles may be made readily for the case of simple harmonic motion, as illustrated in Fig. 5. Successful measurements have been made of the work input with laboratory stirring devices of this type by Krause Ignacio (Master's thesis M. I. T. 1943) with electrical strain gages.

In reciprocating motion the relative velocity of blade and liquid is changing continuously throughout a cycle. In order to obtain values for the mean velocity gradient, therefore, it is necessary to compute the mean value of the dissipation function throughout a cycle.

The basic equations for distance traveled, vertical velocity and vertical acceleration at any point of the stroke in reciprocating motion as illustrated in Fig. 5 are as follows:

Vertical distance traveled:

$$s = r - r\cos\theta \tag{12}$$

Vertical velocity of paddles

$$v = \frac{ds}{dt} = r\sin\theta\,\frac{d\theta}{dt} = 2\pi r s_s \sin\theta \tag{13}$$

Vertical acceleration of paddles:

$$a = \frac{dv}{dt} = 4\pi^2 r S_s^{\,2} \cos\theta \tag{14}$$

where S_s = speed of shaft in RPS

The work per cycle by the drag forces and the dissipation function are given by the following equations:

Work by drag per cycle:

$$\begin{aligned} W_c &= 2\int_0^{\pi} F_D v \frac{d\theta}{2\pi S_s} = 4\pi^2 \frac{62.4}{32.2} C_D \Sigma A r^3 S_s^{\,2} \int_0^{\pi} \sin^3\theta \, d\theta \\ &= \frac{2}{3}\pi^2 \frac{62.4}{32.2} C_D \Sigma A D^3 S_s^{\,2} = 12.74\ C_D \Sigma A D^3 S_s^{\,2} \end{aligned} \tag{15}$$

Dissipation function:

$$W = \frac{W_c S_s}{V} = \frac{12.74 C_D D^3 S_s^{\,3}}{V} \Sigma A \tag{16}$$

It will be noted in equations (15) and (16) that no allowance has been made for movement of the water surrounding the blades. It has been assumed that the average relative velocity between blades and surrounding water is the same as the average relative velocity between the blades and the chamber.

In Ignacio's work with a reciprocating laboratory mixer (previous reference), it was found that work was being done on the mixer by the liquid at the end of the strokes. This was doubtless due to the inertia effect of liquid masses maintaining the velocity imparted by the mixer until after the mixer reversed its stroke. The resulting effect of the liquid inertia forces was to increase somewhat the total work done per cycle, particularly at the higher speeds of the mixer.

The actual work done per cycle was measured in Ignacio's work and the value of the drag coefficient was computed for each run by means of equation (15). For isolated blades in still water of the shape used by Ignacio, the drag coefficient should have a value of 1.5 to 2. The experimental values of the drag coefficient ranged from about 3 to 5.2, the values of the Reynolds number ranging from 150 to 700. The values of the Reynolds number were based on the blade widths and the mean velocity of the blades with respect to the container throughout the stroke. If it be assumed that the correct value for the drag coefficient was 2, that the apparent values found in the experiments were due to higher relative velocities between the mixer and the water, the actual relavtive velocities should be from 20% greater than the assumed average velocity at the lower speed to 60% greater at the higher speed.

The mixing device used by Ignacio in his experiments was a grid system which occupied about 11% of the total volume of the water in the container. Undoubtedly part of the reason for the high values for the drag coefficients was due to the large displacement volume of the mixer. The relative displacement volume for mixing blades in water and sewage treatment plants

would be by comparison negligibly small. For design purposes, therefore, a drag coefficient of about 3 for flat blades should give approximately correct values of the dissipation function when used in equation (16).

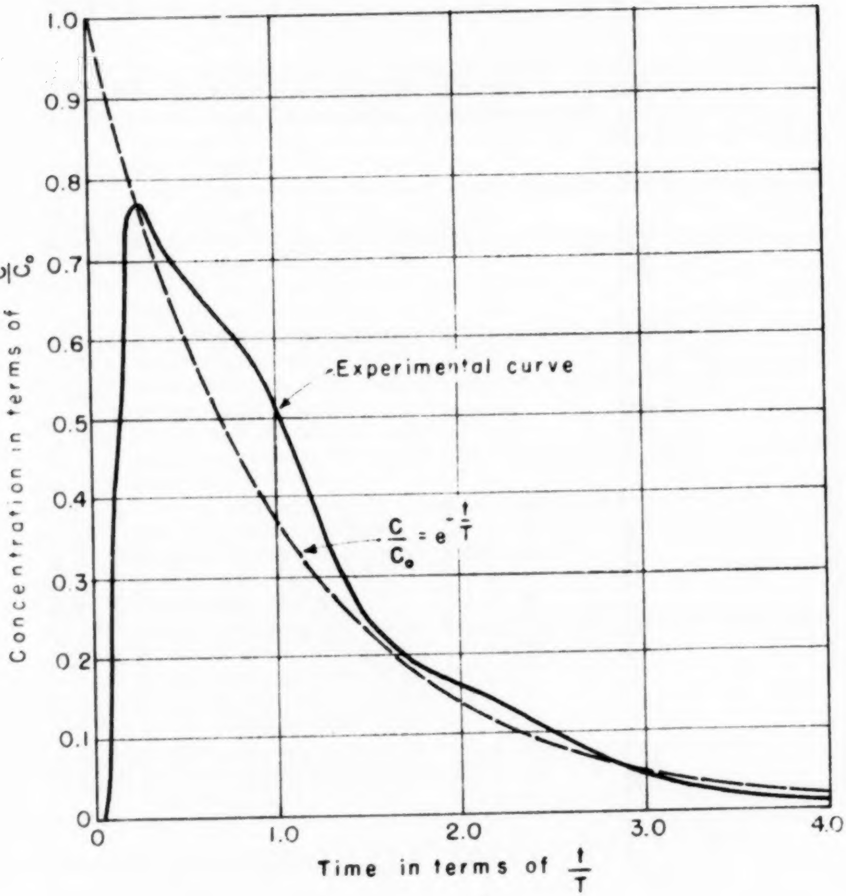
APPENDIX

TABLE 3. DEFINITION OF SYMBOLS

A	=	the cross sectional area of the submerged object perpendicular to the direction of motion with respect to the submerging fluid.
a	=	vertical acceleration of reciprocating paddles.
C	=	concentration of tracer at outlet of tank after time t from introduction at inlet.
C_0	=	initial concentration of tracer in tank if instantaneously dispersed.
C_D	=	the drag coefficient
D	=	diameter of air bubble in Stokes' law, also length of stroke of paddles in reciprocating motion.
e	=	base of Napierian logarithms.
F_D	=	drag force.
G	=	root-mean-square velocity gradient in sec^{-1}
G'	=	velocity gradient at a point in sec^{-1}
G'_{max}	=	maximum value of velocity gradient in fluid field in sec^{-1}
g	=	acceleration of gravity.
γ	=	unit weight of fluid
H	=	depth to diffusers in aeration tank.
h_f	=	head lost by friction.
k	=	ratio of rotating velocity of fluid to rotating velocity of blades.
μ	=	absolute viscosity of fluid.
n	=	number of tanks of equal volume in series.
Q	=	rate of discharge of fluid
Q_a	=	quantity of free air in cfs.
r	=	distance of center of rotating blade from center of shaft, also radius of simple harmonic motion for reciprocating blades.
ρ	=	mass density of fluid.
ρ_1	=	mass density of air in bubble.
S_s	=	speed of rotating shaft in RPS
s	=	vertical distance traveled by reciprocating blades in time t.
T	=	retention period in tank or conduit.
t	=	time of appearance of tracer in concentration C at tank outlet, also time required for reciprocating blades to travel vertical distances.
θ	=	angle of rotation corresponding to vertical distance traveled, s, in simple harmonic motion.
τ	=	unit shearing force in fluid caused by velocity gradient.
V	=	volume of liquid in tank, series of tanks or conduit.
v	=	relative velocity of submerged object with respect to fluid.
W	=	dissipation function of the work of shear in a fluid per unit volume per unit time.
W_c	=	the work of shear in a fluid by reciprocating motion per unit volume per cycle.
Σ	=	the angular velocity in simple harmonic motion.

TABLE 1. FLOCCULATION BASINS IN WATER TREATMENT PLANTS

Plant	Capacity, in million gallons daily	Date built	Type of agitation	Velocity, in ft per second	Retention period, in minutes	Head loss for mixing, in feet	HP per mgd	W ft lbs per sec per cu ft	G sec^{-1}	GT (T in sec)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Albany, N. Y. (Helderberg)	40	1932	Over and under, around end baffles	0.9	20	3		0.156	74	88,800
Atlanta, Ga.	42	1923	Around end baffles	2.0	25	1		0.041	39	58,500
Baltimore, Md. (new)	112	1928	Around end baffles	1.3	30	3		0.104	61	110,000
Cleveland Ohio (Division Avenue)	140	1918	Over and under baffles	0.4	39	0.4		0.011	20	46,800
Denver, Colo.	60	1924	Around end baffles	0.3	20	0.5		0.026	31	37,200
Detroit, Mich. (Springwells)	272	1931	Around end baffles	0.4	17	0.5±		0.030	33	33,600
Erie, Pa. (Chestnut Street)	37	1925	Vertical baffles	0.1	19	1.3		0.068	50	57,000
Flint, Mich.	28	1924	Over and under baffles	0.6	35	1.2		0.036	36	75,600
Fort Worth, Tex.	20	1923	Around end baffles	0.5	40	1±		0.026	31	74,400
Grand Rapids, Mich.	40	1924	Around end baffles	1.0	45	1.0		0.023	29	78,000
Kansas City, Kans.	25	1927	Around end baffles	1.1	10	0.4		0.041	39	23,400
Kansas City, Mo.	100	1928	Spiral flow	---	30	3.5		0.121	67	121,000
Knoxville, Tenn.	15	1927	Mechanical agitators	1.5	20	(6 hp)	0.4	0.120	66	79,200
Miami, Fla.	20	1926	Mechanical agitators	1.0	20	(3 hp)	0.15	0.045	40	48,000
New Orleans, La.	(40 (72	1909) 1928)	Around end baffles	0.8	60	1±		0.017	25	90,000
Oakland, Calif.	12	1927	Mechanical agitators	1.7	20	(4 hp)	0.33	0.098	60	72,000
Oklahoma City, Okla.	16	1923	Over and under baffles	0.6	30	2		0.071	51	92,000
St. Louis, Mo. (Howard Bend)	80	1929	Spiral flow	0.7	16	0.5±		0.032	34	32,600
Tampa, Fla.	14	1926	Mechanical agitators	0.8	100	(8 hp)	0.57	0.034	35	210,000
Washington, D. C.	80	1927	Around end baffles	1.5	27	0.5 to 2		0.077	53	86,000



Concentration in terms of $\frac{C}{C_o}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0 2
0.1
0
Experimental curve
$\frac{C}{C_o} = e^{-\frac{t}{T}}$
0
1.0
2.0
3.0
4.0
Time in terms of $\frac{t}{T}$

FIG. 1

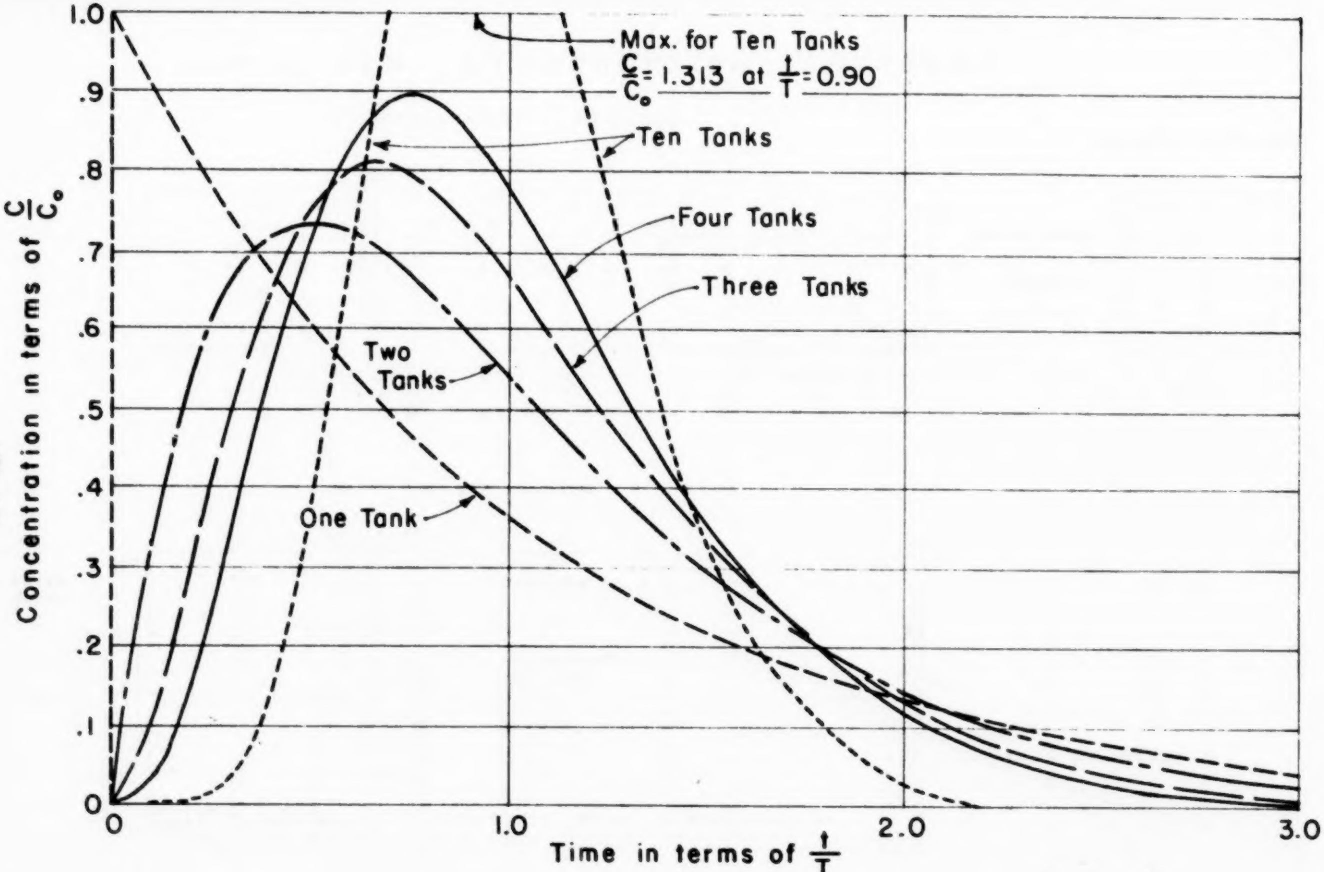

Max. for Ten Tanks
$\frac{C}{C_o} = 1.313$ at $\frac{t}{T} = 0.90$
Ten Tanks
Four Tanks
Three Tanks
Two Tanks
One Tank
Concentration in terms of $\frac{C}{C_o}$
Time in terms of $\frac{t}{T}$
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0
1.0
2.0
3.0

FIG. 2

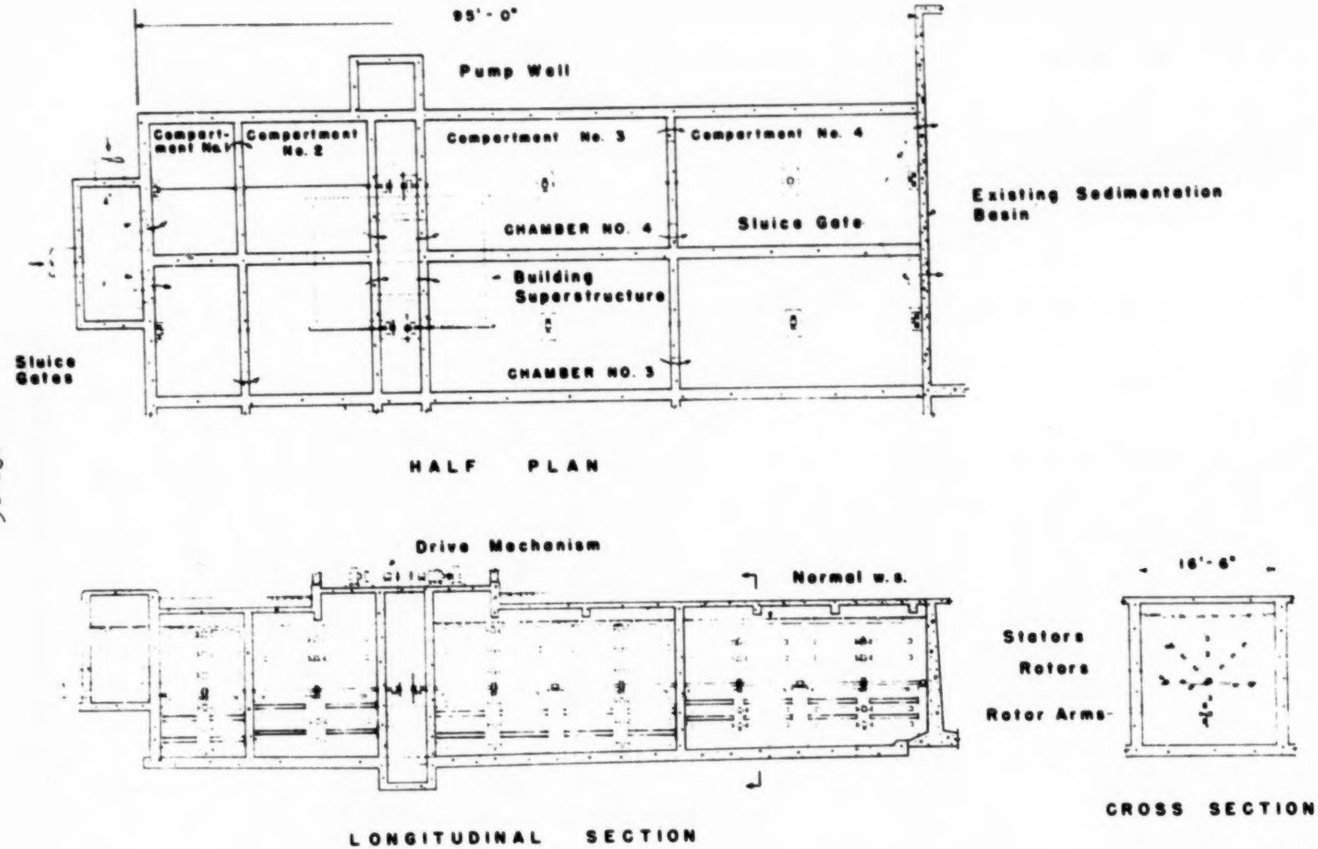
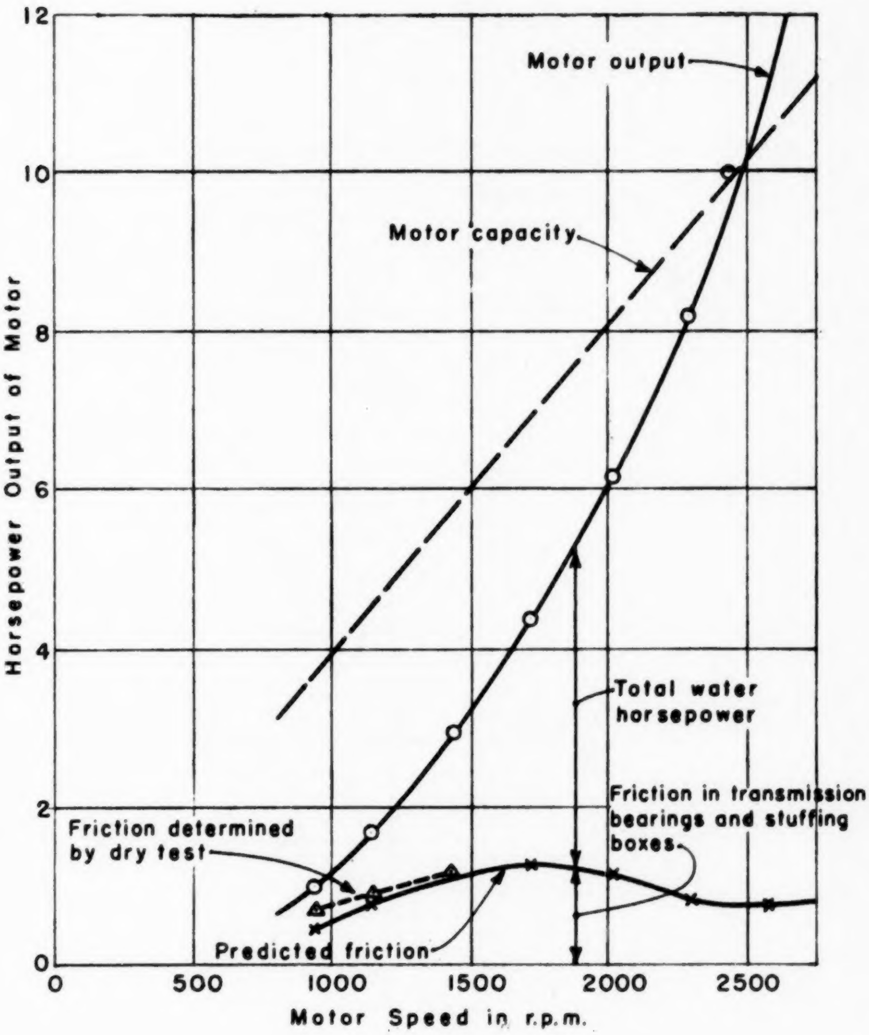


FIG. 3 NEW FLOCCULATION CHAMBERS



Motor output
Motor capacity
Total water horsepower
Friction in transmission bearings and stuffing boxes
Friction determined by dry test
Predicted friction
Horsepower Output of Motor
Motor Speed in r.p.m.
0
2
4
6
8
10
12
0
500
1000
1500
2000
2500

FIG. 4

Angular velocity $\omega = \frac{d\theta}{dt} = 2\pi S_s$

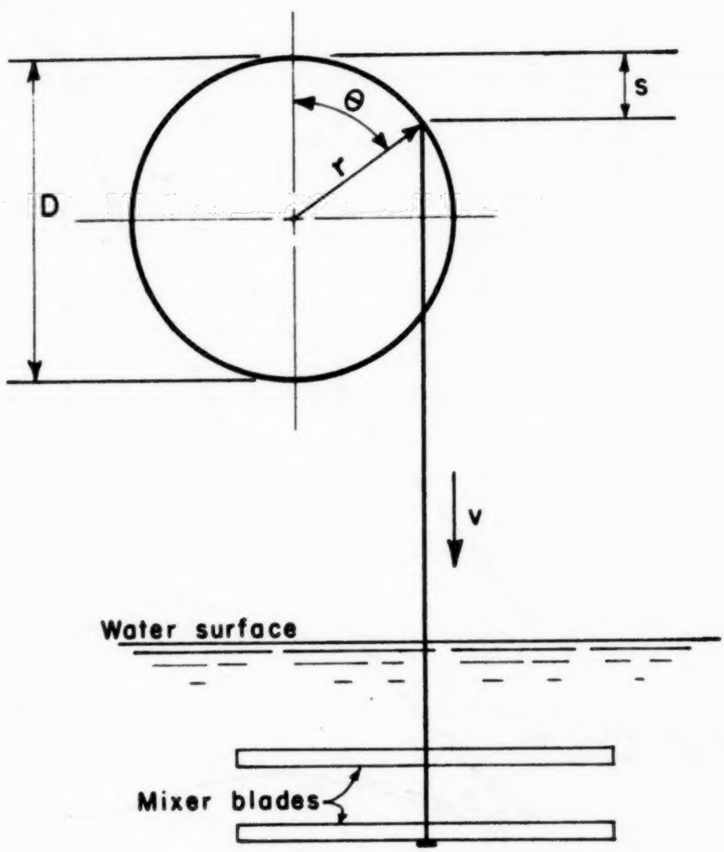


FIG. 5